

PARTICLE PHYSICS

HOMEWORK 6

Halzen and Martin 6.1

(1) Consider $\bar{u}_f i\sigma^{\mu\nu} (p_f - p_i)_\nu u_i = -\frac{1}{2} \bar{u}_f (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) (p_f - p_i)_\nu u_i$

$$\begin{aligned} &= -\frac{1}{2} \bar{u}_f [\gamma^\mu \gamma^\nu p_{f\nu} - \gamma^\nu \gamma^\mu p_{f\nu} - \gamma^\mu \gamma^\nu p_{i\nu} + \gamma^\nu \gamma^\mu p_{i\nu}] u_i \\ &= -\frac{1}{2} \bar{u}_f [2g^{\mu\nu} p_{f\nu} - 2\gamma^\nu \gamma^\mu p_{f\nu} + 2g^{\mu\nu} p_{i\nu} - 2\gamma^\nu \gamma^\mu p_{i\nu}] u_i \\ &= -\frac{1}{2} \bar{u}_f [2(p_f + p_i)^\mu - 2\not{p}_f \gamma^\mu - 2\not{p}_i \gamma^\mu] u_i \\ &= -\bar{u}_f [(p_f + p_i)^\mu - 2m \gamma^\mu] u_i \end{aligned}$$

Solving for $\bar{u}_f \gamma^\mu u_i$ we get

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m} \bar{u}_f [(p_f + p_i)^\mu + i\sigma^{\mu\nu} (p_f - p_i)_\nu] u_i$$

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HALZEN & MARTIN 6.2

$$T_{fi} = -2\pi i \delta(E_f - E_i) \int d^3x \left(\frac{-e}{2m} \right) \bar{u}_f \left[(p_f + p_i)_\mu + i \sigma_{\mu\nu} (p_f - p_i)^\nu \right] u_i \times e^{-i(\vec{p}_f - \vec{p}_i) \cdot \vec{x}}$$

(we get this from HM 6.4, and ~~the~~ the integral over time)

In the non-relativistic limit $(p_f + p_i)_0 \approx 2m$

$(p_f + p_i)_j \approx 0$ (when compared to $2m$)

$(p_f - p_i)^0 \approx 0$ (when compared to $\vec{p}_f$, since the expansion is quadratic)

Then we can write

$$T_{fi} = -2\pi i \delta(E_f - E_i) \int d^3x \left[\bar{u}_f u_i (-eA^0) + \left(\frac{-e}{2m} \right) \bar{u}_f i \sigma_{\mu j} (p_f - p_i)^j u_i \right] e^{-i(\vec{p}_f - \vec{p}_i) \cdot \vec{x}}$$

The first term goes like

$$\bar{u}_f u_i = \bar{u}_F^{(s)} u_I^{(r)} = u_F^{+ (s)} \gamma^0 u_I^{(r)} = 2m \begin{pmatrix} \chi^{(s)\dagger} & 0 \end{pmatrix} \begin{pmatrix} \chi^{(r)} \\ 0 \end{pmatrix} = 2m \delta_{rs}$$

~~$= 2m \delta_{rs}$~~

↑ non relativistic approximation, equation 6.11

This corresponds to the interaction with a potential A^0 (electric potential), i.e. the interaction due to charge only

Let us look at the second term

$$T_{fi}^{(2)} = -2\pi i \delta(E_F - E_i) \int d^3x \left(-\frac{e i}{2m}\right) e^{-i(\vec{p}_F - \vec{p}_i) \cdot \vec{x}} A^\mu \bar{u} \sigma_{\mu j} (p_F - p_i)^j u_i$$

$$\text{But } -i (p_F - p_i)^j e^{-i(\vec{p}_F - \vec{p}_i) \cdot \vec{x}} = \partial^j \left[e^{-i(\vec{p}_F - \vec{p}_i) \cdot \vec{x}} \right]$$

So

$$T_{fi}^{(2)} = -2\pi i \delta(E_F - E_i) \int d^3x \frac{e}{2m} \partial^j \left[e^{-i(\vec{p}_F - \vec{p}_i) \cdot \vec{x}} \right] A^\mu(\vec{x}) \bar{u} \sigma_{\mu j} u_i$$

Integrate by parts -

$$T_{fi}^{(2)} = +2\pi i \delta(E_F - E_i) \int d^3x \frac{e}{2m} e^{-i(\vec{p}_F - \vec{p}_i) \cdot \vec{x}} \partial^j A^\mu(\vec{x}) \bar{u} \sigma_{\mu j} u_i$$

$$T_{fi}^{(2)} = 2\pi i \delta(E_F - E_i) \int d^3x \frac{e}{2m} \partial^j A^\mu(\vec{x}) \bar{\psi}_F \sigma_{\mu j} \psi_i$$

$$\text{Where } \psi \approx u e^{-i\vec{p} \cdot \vec{x}}$$

$$\text{In the non-relativistic limit } \psi \approx \begin{pmatrix} \psi_A \\ 0 \end{pmatrix}$$

Consider

$$\sigma_{ij}^0 = \frac{i}{2} (\gamma^0 \gamma^i - \gamma^i \gamma^0) = \frac{i}{2} \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} - \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

$$\sigma_{ij}^0 = \frac{i}{2} \left[\begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} - \begin{pmatrix} 0 & -\sigma_j \\ -\sigma_j & 0 \end{pmatrix} \right] = i \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}$$

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Then

$$\begin{aligned}\bar{\psi}_F \sigma^0 \psi_I &= \begin{pmatrix} \bar{\psi}_{AF} & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \begin{pmatrix} \psi_{AI} \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & \sigma_j \bar{\psi}_{AF} \end{pmatrix} \begin{pmatrix} \psi_{AF} \\ 0 \end{pmatrix} = 0\end{aligned}$$

So our expression for $T_{fi}^{(2)}$ reduces to

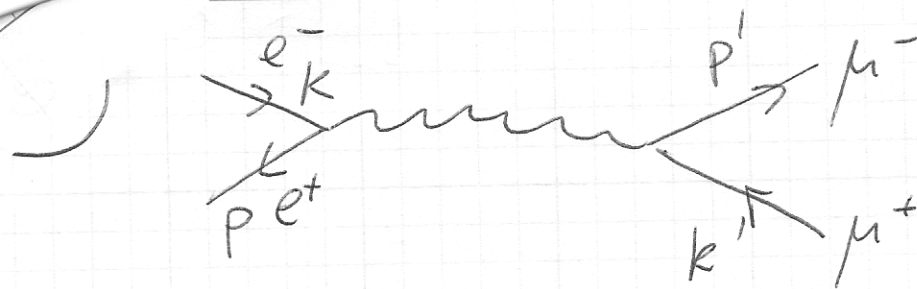
$$T_{fi}^{(2)} = 2\pi i \delta(E_F - E_i) \int d^3x \frac{e}{2m} \partial^\mu A^k(\vec{x}) \bar{\psi}_{AF} \sigma_{kj} \psi_{AI}$$

$$T_{fi}^{(2)} = 2\pi i \delta(E_F - E_i) \int d^3x \frac{e}{2m} \sum_{j>k} (\partial^\mu A^k - \partial^k A^\mu) \psi_{AF}^\dagger (\sigma_{kj}) \psi_{AI}$$

$$\bar{\psi}_{AF} = \psi_{AF}^\dagger$$

$$T_{fi}^{(2)} = 2\pi i \delta(E_F - E_i) \int d^3x \frac{e}{2m} \vec{B} \cdot \vec{\sigma} \psi_{AF}^\dagger \psi_{AI}$$

(using the hint in the book)



$$q = k + p$$

$$M = -\frac{e^2}{q^2} [\bar{v}(p) \gamma^\mu u(k)] [\bar{u}(p') \gamma_\mu v(k')]$$

$$|M|^2 = \frac{e^4}{q^4} L_{(electron)}^{\mu\nu} L_{\mu\nu}^{(muon)}$$

$$L_{(electron)}^{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} [\bar{v}(p) \gamma^\mu u(k)] [\bar{u}(k) \gamma^\nu v(p)]$$

Write down indices explicitly

$$L_{(electron)}^{\mu\nu} = \frac{1}{2} \sum \bar{v}_\alpha(p) \gamma_\alpha^\mu u_\beta(k) \bar{u}_\gamma(k) \gamma_\gamma^\nu v_\delta(p)$$

$$L_{(electron)}^{\mu\nu} = \frac{1}{2} (\not{p} - m)_{\alpha\gamma} \gamma_\alpha^\mu (\not{k} + m)_{\beta\delta} \gamma_\gamma^\nu$$

$$L_{(electron)}^{\mu\nu} = \frac{1}{2} \text{Trace} [(\not{p} - m) \gamma^\mu (\not{k} + m) \gamma^\nu]$$

ultrarelativistic, $m \approx 0$

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$$L^{\mu\nu}_{\text{electron}} = \frac{1}{2} \text{Trace} [\not{p} \gamma^\mu \not{k} \gamma^\nu] =$$

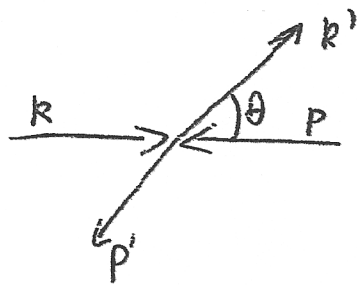
$$L^{\mu\nu}_{\text{electron}} = 2 [p^\mu k^\nu + p^\nu k^\mu - g^{\mu\nu} (pk)]$$

$$\text{Similarly } L^{\mu\nu}_{\text{(muon)}} = 2 [p'_\mu k'_\nu + p'_\nu k'_\mu - g_{\mu\nu} (p'k')]$$

$$|\overline{M}|^2 = \frac{4e^4}{q^4} [p^\mu k^\nu + p^\nu k^\mu - g^{\mu\nu} (pk)] [p'_\mu k'_\nu + p'_\nu k'_\mu - g_{\mu\nu} (p'k')]$$

$$|\overline{M}|^2 = \frac{4e^4}{q^4} [(pp')(kk') + (pk')(kp') - (pk)(p'k') + (pk')(kp') + (pp')(kk') - (pk)(p'k') + (pk)(p'k') - (pk)(p'k') + 4(pk)(p'k')]$$

$$|\overline{M}|^2 = \frac{8e^4}{q^4} [(pp')(kk') + (pk')(kp')]$$



$$|\vec{p}| = |\vec{k}| = |\vec{p}'| = |\vec{k}'| = E$$

$$-pp' = E^2(1 - \cos\theta) \quad kk' = E^2(1 - \cos\theta) \quad pk' = E^2(1 + \cos\theta) \quad (p'k) = E^2$$

$$q^4 = (k+p)^4 = (2E)^4 = 16E^4$$

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$$|\overline{M}|^2 = \frac{8e^4}{16E^4} \left[E^4 (1 - \cos\theta)^2 + E^4 (1 + \cos\theta)^2 \right]$$

$$|\overline{M}|^2 = \frac{8e^4}{2} \left[1 + \cos^2\theta - 2\cos\theta + 1 + \cos^2\theta + 2\cos\theta \right] = e^4 (1 + \cos^2\theta)$$

$$\left. \frac{d\sigma}{d\Omega} \right|_{CM} = \frac{1}{64\pi^2 s} |\overline{M}|^2 = \frac{e^4}{64\pi^2 s} (1 + \cos^2\theta)$$

$$\text{But } \alpha = \frac{e^2}{4\pi} \Rightarrow e^4 = 16\alpha^2 \pi^2$$

$$\Rightarrow \left. \frac{d\sigma}{d\Omega} \right|_{CM} = \frac{\alpha^2}{4s} (1 + \cos^2\theta)$$

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(i) Figure 6.9, θ is the angle between the incoming electron and the outgoing fermion. In our case, the b -

(ii)

$$\frac{d\sigma}{d\Omega} (\bar{e}_L e_R^+ \rightarrow \bar{b}_L b_R) = 3 \frac{\alpha^2}{4s} (1 + \cos\theta)^2 \left[\frac{1}{3} + r c_L^b c_L^e \right]^2$$

$$\frac{d\sigma}{d\Omega} (\bar{e}_L e_R^+ \rightarrow b_R \bar{b}_L) = 3 \frac{\alpha^2}{4s} (1 - \cos\theta)^2 \left[\frac{1}{3} + r c_R^b c_L^e \right]^2$$

$$\frac{d\sigma}{d\Omega} (\bar{e}_R e_L^+ \rightarrow b_R \bar{b}_L) = 3 \frac{\alpha^2}{4s} (1 + \cos\theta)^2 \left[\frac{1}{3} + r c_R^b c_R^e \right]^2$$

$$\frac{d\sigma}{d\Omega} (\bar{e}_R e_L^+ \rightarrow b_L \bar{b}_R) = 3 \frac{\alpha^2}{4s} (1 - \cos\theta)^2 \left[\frac{1}{3} + r c_L^b c_R^e \right]^2$$

To get the unpolarized cross-section, I need to sum all four and divide by 4

let's calculate the terms in square brackets - Careful, r is complex

$$\left[\frac{1}{3} + r c_L^b c_L^e \right]^2 = \frac{1}{9} + |r|^2 c_L^{b2} c_L^{e2} + \frac{2}{3} c_L^b c_L^e \text{Re}(r)$$

$$\left[\frac{1}{3} + r c_R^b c_L^e \right]^2 = \frac{1}{9} + |r|^2 c_R^{b2} c_L^{e2} + \frac{2}{3} c_R^b c_L^e \text{Re}(r)$$



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$$\left[\frac{1}{3} + r C_R^b C_R^e\right]^2 = \frac{1}{9} + |r|^2 C_R^{b^2} C_R^{e^2} + \frac{2}{3} C_R^b C_R^e \operatorname{Re}(r)$$

$$\left[\frac{1}{3} + r C_L^b C_R^e\right] = \frac{1}{9} + |r|^2 C_L^{b^2} C_R^{e^2} + \frac{2}{3} C_L^b C_R^e \operatorname{Re}(r)$$

$$\begin{aligned} \text{So } \frac{d\sigma}{d\Omega} = \frac{3\alpha^2}{16s} & \left\{ [1 + \cos\theta]^2 \left[\frac{2}{9} + |r|^2 (C_L^{b^2} C_L^{e^2} + C_R^{b^2} C_R^{e^2}) + \frac{2}{3} \operatorname{Re}(r) [C_L^b C_L^e + C_R^b C_R^e] \right. \right. \\ & \left. \left. + \frac{3\alpha^2}{16s} \left\{ [1 - \cos\theta]^2 \left[\frac{2}{9} + |r|^2 (C_R^{b^2} C_L^{e^2} + C_L^{b^2} C_R^{e^2}) + \frac{2}{3} \operatorname{Re}(r) [C_R^b C_L^e + C_L^b C_R^e] \right. \right. \right. \right. \end{aligned}$$

Now Table 13.2

$$C_A^b = -\frac{1}{2}$$

$$C_V^b = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w$$

$$C_A^e = -\frac{1}{2}$$

$$C_V^e = -\frac{1}{2} + 2 \sin^2 \theta_w$$

$$C_L^b = C_V^b + C_A^b = -1 + \frac{2}{3} \sin^2 \theta_w$$

$$C_L^e = C_V^e + C_A^e = -1 + 2 \sin^2 \theta_w$$

$$C_R^b = C_V^b - C_A^b = \frac{2}{3} \sin^2 \theta_w$$

$$C_R^e = C_V^e - C_A^e = 2 \sin^2 \theta_w$$

~~$$\frac{3\alpha^2}{16s} \left\{ [1 + \cos\theta]^2 \left[\frac{2}{9} + |r|^2 \left(\left(1 + \frac{4}{9} \sin^4 \theta_w - \frac{4}{3} \sin^2 \theta_w\right) (1 + 4 \sin^4 \theta_w - 4 \sin^2 \theta_w) \right) + \frac{2}{3} \operatorname{Re}(r) \right] \right. \right.$$~~

Let's calculate the various terms; use shorthand $\boxed{s = \sin^2 \theta_w}$

$$\begin{aligned} C_L^{b^2} C_L^{e^2} + C_R^{b^2} C_R^{e^2} &= \left(\frac{2}{3}s - 1\right)^2 (2s - 1)^2 + \frac{4}{9} s^2 4s^2 \\ &= \left(\frac{4}{9}s^2 + 1 - \frac{4}{3}s\right) (4s^2 - 4s + 1) + \frac{16}{9} s^4 \\ &= \frac{16}{9} s^4 + \frac{16}{9} s^3 + \frac{4}{9} s^2 + 4s^2 - 4s + 1 - \frac{16}{3} s^3 + \frac{16}{3} s^2 - \frac{4}{3} s + \frac{16}{9} s^4 \end{aligned}$$

~~$$= \frac{32}{9} s^4 + \frac{8}{3} s^3 + \frac{20}{9} s^2 - \frac{4}{3} s + 1$$~~

(10)

$$C_L^{b^2} C_L^{e^2} + C_R^{b^2} C_R^{e^2} = \frac{\frac{32}{9}s^4 - \frac{16+48}{9}s^3 + \frac{4+36+48}{9}s^2 - \frac{4+12}{3}s + 1}{\cancel{\frac{32}{9}s^4 - \frac{64}{9}s^3 + \frac{88}{9}s^2 - \frac{16}{3}s + 1}} = \frac{32}{9}s^4 - \frac{64}{9}s^3 + \frac{88}{9}s^2 - \frac{16}{3}s + 1$$

$$C_L^b C_L^e + C_R^b C_R^e = \left(\frac{2}{3}s - 1\right)(2s - 1) + \frac{4}{3}s^2$$

$$= \frac{4}{3}s^2 - \frac{2+6}{3}s + 1 + \frac{4}{3}s^2 = \frac{8}{3}s^2 - \frac{8}{3}s + 1$$

$$C_R^{b^2} C_L^{e^2} + C_L^{b^2} C_R^{e^2} = \frac{4}{9}s^2(2s-1)^2 + \left(\frac{2}{3}s-1\right)^2 4s^2$$

$$= \frac{4}{9}s^2(4s^2-4s+1) + 4s^2\left(\frac{4}{9}s^2 - \frac{4}{3}s + 1\right)$$

$$= \frac{16}{9}s^4 - \frac{16}{9}s^3 + \frac{4}{9}s^2 + \frac{16}{9}s^4 - \frac{16}{3}s^3 + 4s^2$$

$$= \frac{32}{9}s^4 - \frac{64}{9}s^3 + \frac{40}{9}s^2$$

$$C_R^b C_L^e + C_L^b C_R^e = \frac{2}{3}s(2s-1) + \left(\frac{2}{3}s-1\right)2s$$

$$= \frac{4}{3}s^2 - \frac{2}{3}s + \frac{4}{3}s^2 - 2s = \frac{8}{3}s^2 - \frac{8}{3}s$$

So..... for $\frac{d\sigma}{d\Omega}$ I will have

$$\frac{d\sigma}{d\Omega} = \frac{3d^2}{16s} [A_0(1+\cos^2\theta) + A_1\cos\theta]$$

$$\text{with } A_0 = \frac{4}{9} + |r|^2 \left[\frac{32}{9}s^4 - \frac{64}{9}s^3 + \frac{88}{9}s^2 - \frac{16}{3}s + 1 + \frac{32}{9}s^4 - \frac{64}{9}s^3 + \frac{40}{9}s^2 \right]$$

$$+ \frac{2}{3} \text{Re}(r) \left[\frac{8}{3}s^2 - \frac{8}{3}s + 1 + \frac{8}{3}s^2 - \frac{8}{3}s \right]$$

(17)

$$A_0 = \frac{4}{9} + |r|^2 \left[\frac{64}{9} s^4 - \frac{128}{9} s^3 + \frac{128}{9} s^2 - \frac{16}{3} s + 1 \right] + \frac{2}{3} \operatorname{Re}(r) \left[\frac{16}{3} s^2 - \frac{16}{3} s + 1 \right]$$

$$A_1 = 2 \left[\frac{4}{9} + |r|^2 \left(\frac{32}{9} s^4 - \frac{64}{9} s^3 + \frac{88}{9} s^2 - \frac{16}{3} s + 1 \right) + \frac{2}{3} \operatorname{Re}(r) \left(\frac{8}{3} s^2 - \frac{8}{3} s + 1 \right) \right]$$

$$A_1 = 2 |r|^2 \left[\frac{32}{9} s^4 - \frac{64}{9} s^3 + \frac{88}{9} s^2 - \frac{16}{3} s + 1 - \frac{32}{9} s^4 + \frac{64}{9} s^3 - \frac{40}{9} s^2 \right] + \frac{4}{3} \operatorname{Re}(r) \left[\frac{8}{3} s^2 - \frac{8}{3} s + 1 - \frac{8}{3} s^2 + \frac{8}{3} s \right]$$

$$A_1 = 2 |r|^2 \left[\frac{48}{9} s^2 - \frac{16}{3} s + 1 \right] + \frac{4}{3} \operatorname{Re}(r)$$

And the total cross-section

$$\sigma = 2\pi \int \frac{d\sigma}{d\cos\theta} d\cos\theta = 2\pi \frac{3\alpha^2}{16s} \int_{-1}^1 A_0 (1+x^2) dx + 2\pi \frac{3\alpha^2}{16s} \int_{-1}^1 A_1 x dx$$

$$\sigma = \frac{3\pi\alpha^2}{8s} A_0 \left[x + \frac{1}{3} x^3 \right]_{-1}^1 = \frac{3\pi\alpha^2}{8s} A_0 \left[2 + \frac{2}{3} \right] = \frac{\pi\alpha^2}{s} A_0$$

In order to plot the cross-section, I need to get expressions for $|r|^2$ and $\operatorname{Re}(r)$

(12)

$$\frac{\sqrt{2} G M_Z^2}{(s-M_Z^2)+i\Gamma_Z M_Z} \frac{s}{e^2} = \frac{s}{e^2} \sqrt{2} G M_Z^2 \frac{(s-M_Z^2-i\Gamma_Z M_Z)}{(s-M_Z^2)^2-M_Z^2 \Gamma_Z^2}$$

$$r = \frac{s}{4\pi\alpha} \frac{\sqrt{2} G M_Z^2 (s-M_Z^2-i\Gamma_Z M_Z)}{(s-M_Z^2)^2-M_Z^2 \Gamma_Z^2}$$

$$\text{Re}(r) = \frac{s}{4\pi\alpha} \frac{\sqrt{2} G M_Z^2 (s-M_Z^2)}{(s-M_Z^2)^2-M_Z^2 \Gamma_Z^2}$$

$$|r|^2 = \frac{s^2}{16\pi^2\alpha^2} \frac{2 G^2 M_Z^4}{\left([s-M_Z^2]^2 - M_Z^2 \Gamma_Z^2\right)^2} \Rightarrow \frac{s^2 G^2 M_Z^4}{8\pi^2\alpha^2} \frac{1}{(s-M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

And to get it into μb , I will calculate it in GeV^{-2} and then use $(1\text{ GeV})^{-2} = 389 \mu\text{b}$

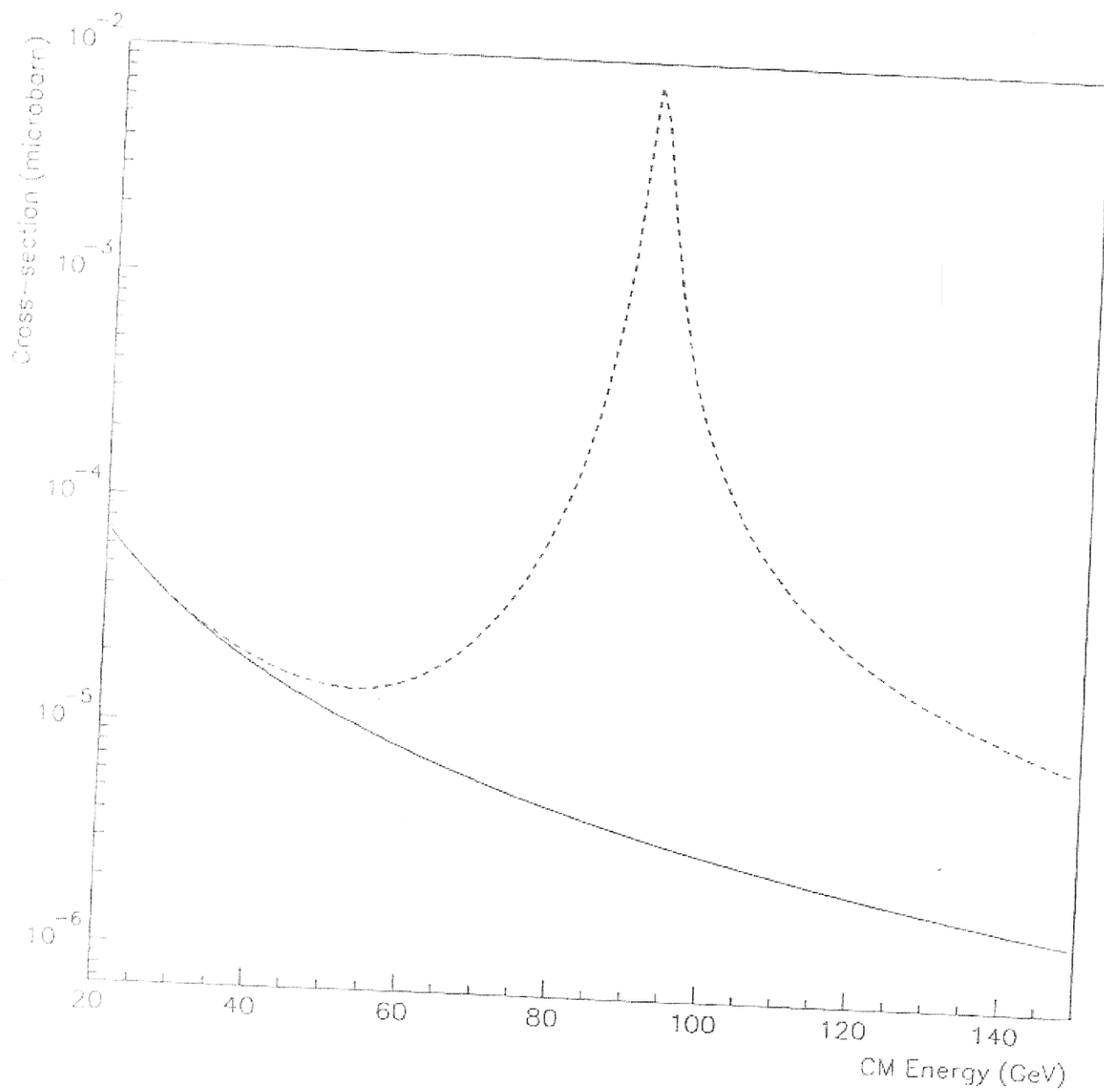
Also, I will need to use $G = 1.16 \cdot 10^{-5} \text{ GeV}^{-2}$

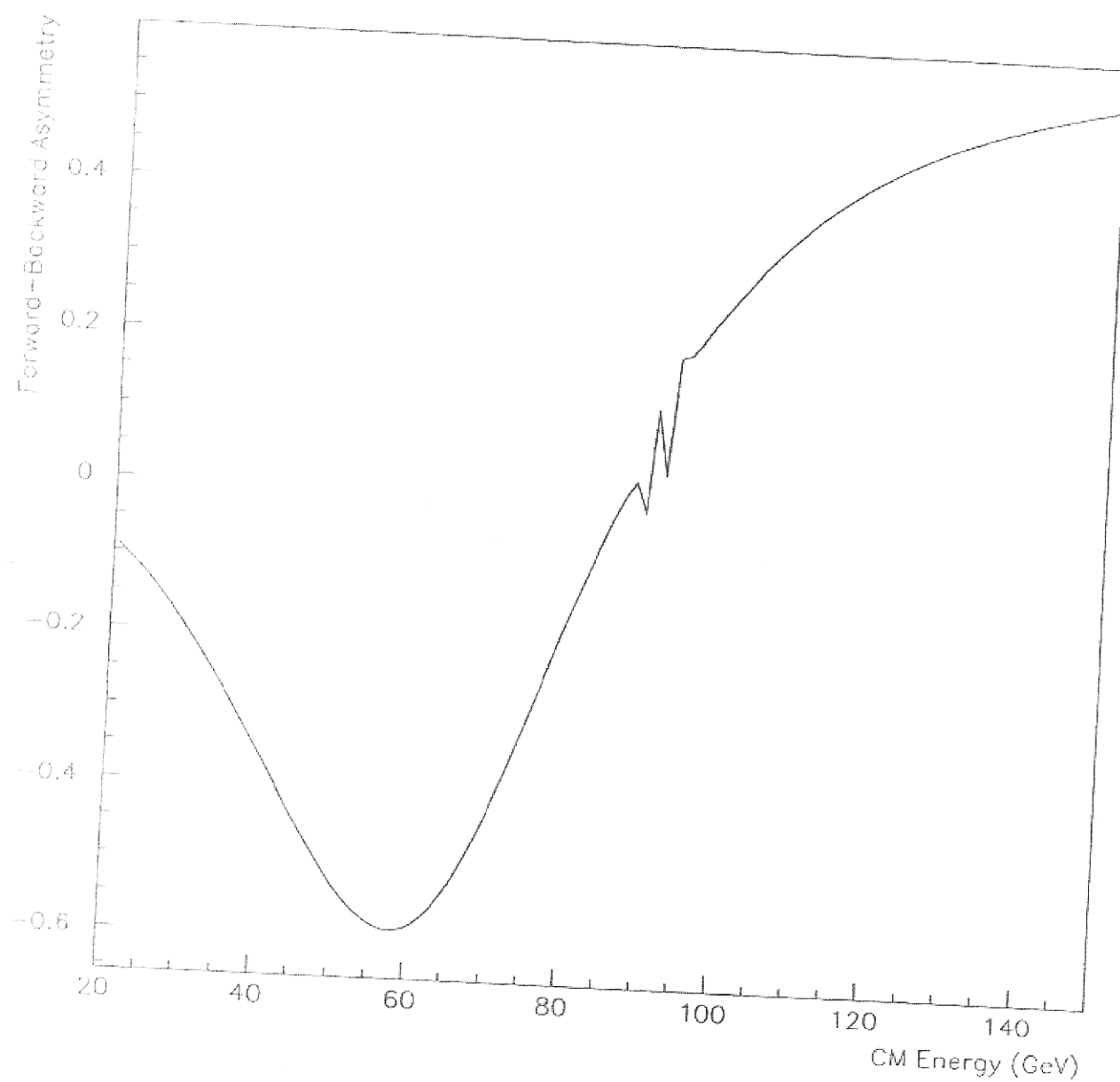
By contrast, the purely QED cross section is simply by replacing A_0 with

$$A_0^{\text{QED}} = \frac{4}{9}$$

The asymmetry is $A_{\text{FB}} = \frac{A_1}{(8A_0/3)}$

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Experimentally

Need not only to identify $b\bar{b}$ events, but also to tag flavor.
Best way is lepton tag (e or μ)

$$b \rightarrow c \ell^- \nu$$

$$\bar{b} \rightarrow c \ell^+ \nu$$

Backgrounds ① $c\bar{c}$ pairs

② sequential $b \rightarrow c \rightarrow \ell$

③ fake leptons (i.e. hadrons that fake lepton signature)

→ beat down the backgrounds

③ good (!) lepton detection

② leptons from $b \rightarrow \ell$ have higher momentum than $b \rightarrow c \rightarrow \ell$

① leptons from $c \rightarrow \ell$ have lower P_T with respect to jet axis than leptons from $b \rightarrow \ell$

①, ③ $\tau_b > \tau_c$ and $\tau_b \gg \tau(\text{non-charmed hadrons})$

$\Rightarrow$ use vertex detector